Dynamics of Interacting Strategies

Masanao Aoki[1] and Hiroyuki Moriya[2]

[1] Department of Economics, University of California, Los Angeles
aoki@econ.ucla.edux
[2] Oxford Financial Education moriya@quasars22.co.jp

Abstract. This papers presents the model of the dynamics process of switching in the strategy adopted by a large number of agents according to their views of what they deem as the most advantageous strategy in relation to the behavior of other agents and/or exogenous environments. The process of switching strategy is modeled by master equation by suitably specifying the transition rates of continuous time Markov chains. The computer simulation explains the effects of demand-supply imbalance created by short-medium term traders in the dollar-yen foreign exchange market.

1 Introduction

We examine nonlinear dynamics generated by a large number of heterogeneous agents when they switch the strategy or go in/out of the strategy. They change the strategy or join/get out the strategy over time, because they can not foresee the consequences of their choices exactly at the moment of their choice. Consequences of their choices are distributed stochastically, and over time new information may become available as to desirability of some choices over the others.

Consequently, clusters of agents of the same choices may develop and disappear over time. Aoki (1996,2002) has discussed problems for the case where each agent has binary choices. As in these cases, we use the master equation, that is, the backward Chapman-Kolmogorov equation, to discuss the dynamics of agent behavior.

In the late 1980s and until the mid 1990s, Hogg and Huberman (1991), Youssefmir and Huberman (1995) or Adjali, Gell and Lunn (1994), and their collaborators have published a number of papers in which agents have many choices over resources and strategies. While these authors use error functions to express distributions of the consequences of choices. We use Ingber's approximation to the error function and introduce Gibbs distributions into transition rates of continuous time Markov chains.

Here, the computer simulations focus on situations that agents implement two strategies in dollar-yen foreign exchange market.

2 The model

Suppose that there are a fixed number K strategies. The total number of agents is fixed at $\mathbf{N}$. At any time n_i is the number of agents with strategy i, where $\sum_i n_i = \mathbf{N}$. The master equation describe how the probability $Pr(\mathbf{n}, t)$ evolves over time, where $\mathbf{n}$ is the vector whose i-th component is n_i. We say agent is of type i when it uses strategy i.

The probability $Pr(\mathbf{n}, t + \Delta)$ increases over $Pr(\mathbf{n}, t)$ by the net inflow of probability flux, that is, the difference between the inflow and outflow, where inflow arise from some agent of type j deciding to drop strategy j and adopting strategy i, $j \neq i$, and outflow is due to one agent of type i deciding to switch to a different strategy.

Since we model those processes as birth-and-death type Markov process, at most one such strategy switch takes place over a small time interval Δ.

The master equation is derived from

$$Pr(\mathbf{n}, t + \Delta) = Pr(\mathbf{n}, t) - \sum_{\mathbf{n}' \neq \mathbf{n}} Pr(\mathbf{n}, t)\omega(\mathbf{n}, \mathbf{n}') + \sum_{\mathbf{n}' \neq \mathbf{n}} Pr(\mathbf{n}', t)\omega(\mathbf{n}', \mathbf{n})$$

Assuming that λ is the rate of strategy examination over time, denoting the number of agents of type j before revision by n'_j, and let $\eta_{j,i}$ the probability that strategy i is regarded by agent j to be the most desirable, we write the transtion probability over time interval Δ as

$$\omega(\mathbf{n}', \mathbf{n}) = n'_j \eta_{j,i}(\mathbf{n}')\Delta$$

Aoki(2002) has shown that on the derivation that η has a Gibbs distribution $e^{\beta g_{j,i}}/Z$, where Z is a partition function, where β is a parameter which embody the uncertainty associated with this switch of strategy, and $g_{j,i}$ is the expected difference in the discounted present value of adopting strategy j over strategy i. Here we use Ingber's approximation to error functions for approximating transition rates in the way described by Aoki (1996,page 133;1998;2002,chap 6). Hence the master equation is rewritten as

$$Pr(\mathbf{n}, t + \Delta) - Pr(\mathbf{n}, t) = \lambda\Delta(O - I),$$

where $O - I$ stands for inflow - outflow, where

$$I = \sum_i \sum_{j \neq i} n'_j \eta_i Pr(\mathbf{n}', t),$$

and

$$O = \sum_i \sum_{j \neq i} n_j \eta_j Pr(\mathbf{n}, t),$$

up to $o(\Delta)$.

Dividing both sides by Δ and letting it go to zero we arrive at

$$\frac{\partial Pr(\mathbf{n}, t)}{\partial t} = \lambda(O - I)$$

3 Interacting or No interacting patterns

Calculating η_i for interacting patterns: Let V_i be the random discounted present value of using strategy $i, i = 1, \ldots, n$ for some specified length of time. Define $\eta_{i,j}$ to be the probability that agents who have been using strategy i want to switch from strategy i to j,

$$\eta_{i,j}(x) = Pr(V_j \geq \max_{i \neq j}\{V_i\}|x)$$

Under certain sets of assumptions, it is known that this expression is given by a Gibbs distribution, Aoki(2002,Sec.6.3). We can use a program called MULNOR, introduced by Shervich(1984,1985), to calculate such probabilities with agents interactions.

Calculating equilibrium probability for no interacting patterns Master equation with entry and exit without any switching among strategies provides the equilibrium probability of the strategy based on Poisson distribution.

$$P(\theta = \nu) = e^{-\theta} \frac{\theta^{\nu}}{\nu!}$$

where $\theta = \alpha/(\mu k)$; α:the number of entry; μk:the number of exit: k:the number of traders.

4 Simulation

The simulation is made for identifying how the behavior of trading groups with the short-medium term horizon affects price movements in the foreign exchange market. We focus on two types of traders in the market: trend followers and contrarians. Trend followers buy(or sell) currency when the currency is appreciating(or depreciating). They are divided into type 1a and type 1b. Type 1a is a upward trend follower who gets profits when the market has upward trend. There are large number of trading strategies for upward trend viewers, we use option strategies to replicate their behavior. The trading with buying calls represents type 1a strategy. Type 1b is a downward trend follower who makes money when the market has downward trend. Buying puts represents type 1b strategy. The trend followers switch the strategy from 1a to 1b or vice versa, depending on their profits and losses. These interactions are described as birth-death process. Type 1a and type 1b have the master equation with transition rates that are function of profits and losses.

Type 2 is a contrarian who buy(or sell) the currency when it is depreciating(or appreciating), because they believe that the market will stay in the range. Selling calls and puts represents type 2 strategy.

We assume that trend followers and contrarians do not change their types in short-medium term horizon. However, they go in/out the strategy over time depending on profits and losses of each strategy. Type 1 and type 2 have the master equations with entry and exit without interacting patterns.

The simulations are made as follows:

1. Trend followers : Trend followers buy one unit(Yen) of at the money call(or put) with one(or three) month(s) maturity every day. They hold the positions until the maturity. Simplifying the problems, one(three) month(s) consists of 20(60) working days. Therefore, the portfolios of options held by each type include 20(60) different options. A set of daily historical data are used for the evaluations: spot price, implied volatilities, and domestic and foreign interest rates. In case of type 1a with one month maturity, the portfolio are evaluated daily as

$$V_{1a}(t) = \sum_{t_p=0}^{20-1} (w_{t-t_p} \times c_{t-t_p,t} - r_{t-t_p})$$

where $c_{t-t_p,t}$ is the value of the call option at time t starting at time $t - t_p$ as the at the money option with maturity of one month, and is evaluated by using the Black-Scholes type currency option model(M.b.Garman and S.W.Kohlhagen,1983). w_{t-t_p} is the weight of the option and equal to the inverse of $c_{t-t_p,t-t_p}$. r_{t-t_p} is the funding cost for the option starting at time $t-t_p$. Based on the standard deviations and means of $V_{1a}(t)$, we estimate the rate, $\eta_{1a,1b}(t)$ by using Mulnor program,

$$\eta_{1a,1b}(x(t)) = Pr(V_{1a} \geq V_{1b}|x(t)) = \int_{-\infty}^{\infty} \int_{a}^{\infty} f(x_{1a}, x_{1b}) dx_{1a} dx_{1b}.$$

The probability of type 1a at time $t + \Delta$ based on the set of empirical data is obtained from

$$P(n_{1a}, t + \Delta) = P(n_{1a}, 0) + \sum_{t=0} P(n_{1a}, t)\omega(n_{1a}, n_{1b}, t), 0 \leq P(n_i, t) \leq 1$$

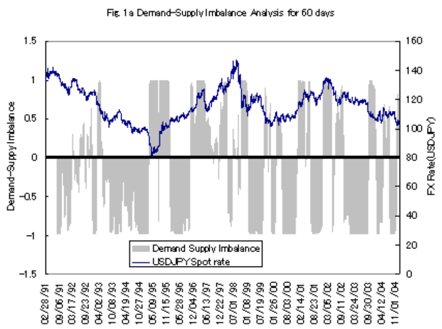
for any t, where $\omega(n_{1a}, n_{1b}, t) = l \times \eta_{1a,1b}(x(t))$. l is constant over time and determined as maximizing the profit of trend followers.

Finally, we get the daily value of the portfolio held by the trend followers.

$$V(t) = P(n_{1a}, t + \Delta) \times V_{1a}(t) + (1 - P(n_{1a}, t + \Delta)) \times V_{1b}(t)$$

2. Contrarians : Contrarians sell one unit(Yen) of at the money call and put with one(or three) month(s) maturity every day and keep these positions until the maturity. The value of the portfolio is calculated by the same way as above.

3. Exit and Entry to the strategy : Finally, we calculate the standard deviation of the portfolio value of both trend followers and contrarians, and estimate the equilibrium probability of each strategy with entry and exit.



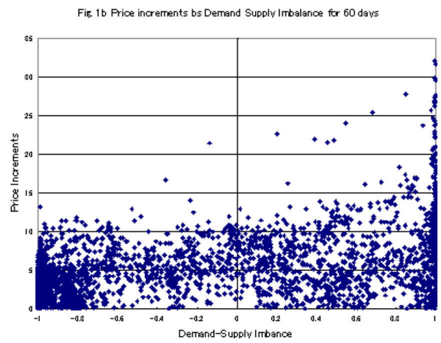
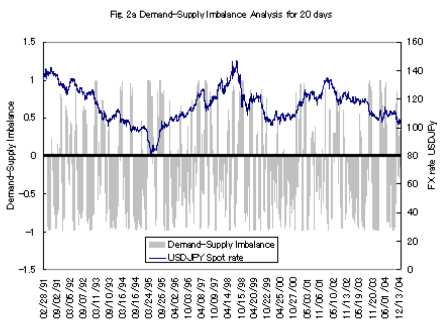


Fig 1a provides the relationship between demand/supply imbalance(60 days) and dollar-yen price movements. Fig 1b provides the relationship between price increments(60 days) and demand/supply imbalance.



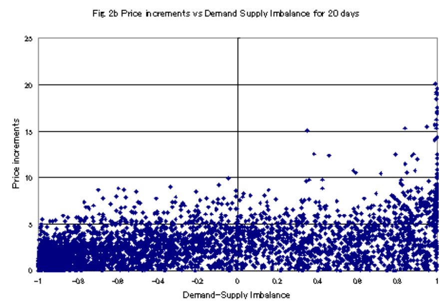


Fig 2a provides the relationship between demand/supply imbalance(20 days) and dollar-yen price movements. Fig 2b provides the relationship between price increments(20 days) and demand/supply imbalance(20 days).

The decision of entry and exit of each group is made independently, therefore, there are the imbalance between demand and supply of currency(options). This imbalance is balanced by market-maker and day traders in the real markets, however, we currently focus on the imbalance created by the short-medium term traders. In general, we can understand when trend followers dominate the market, they will provide the positive feedback of the price movements that emerge the trend in the market. On the other hand, when contrarians dominate the market, the market will stay in the range due to the negative feedback of the price movements. The imbalance at time t is obtained by $P_{n_1}(t) - P_{n_2}(t)$. Fig 1 and 2 show us the simulation results that explain the demand-supply imbalance affects the price movements in dollar-yen market.

5 Conclusion

We examine nonlinear dynamics generated by trend followers and contrarians with short-medium view in the dollar-yen market. Based on the analysis of computer simulation, we currently conclude that behavior of heterogeneous agents may be one of the reasons for generating trending or trendless market.

References

1. Adijali, I,M.Gell, and T. Lunn (1994), "fluctuations in a decentralzied agent-resource system," Phy. Rev. E 49 3833-3842
2. Aoki,M.(1996) New Approaches to Macroeconomic Modeling:Evolutionary stochastic dynamics, multiple equilibria, and externalities as field effects, Cambridge University Press, New York.
3. Aoki,M.(1998) "A simple model of asymmetrical business cycles: Interactive dynamics of large number of agents with discrete choices," Macroeconomic Dynamics,2,427-442.
4. Aoki,M.(2002) Modeling Aggregate Behavior and Fluctuations in Economics, Cambridge University Press, New York.
5. Drezner, Z. (1992), "Computation of the multivariate normal integral," ACM Trans. Math. soft. 18, 470-480.
6. Garman,Mark B. and Kohohagen, Steven W., "Foreigh Currency Option Values", Journal of International Money and Finance,Vol.2,No.3,Dec.1983,231-237
7. Genz, A. (2004), "Numerical computation of rectangular bibariate and trivariate normal and t probabilities," Stat. Comput., 14, 252-260.
8. Hogg, T., and B. A. Huberman (1991) "Controlling chaos in distributed systems," IEEE Trans. Systems, Man, and Cybernetics, 21, 1325-1331.
9. Ingber, L. (1982), "Statistical mechanics of neocortical interactions," Physica A, 5, 83-107.
10. Schervish, M. (1984), "Multivariate normal probabilities with error bound," Appl. Stat., 33, 81-87.
11. Schervish, M. (1985), "Correction to Algorithm AS 195: Multivariate normal probabilities with error bound", Appl. Stat., 34, 103-104.
12. Youssefmir, M., and B. A. Huberman (1995), "Clustered Volatility in Multiagent Dynamics", arX:adap-org/9502006.